ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL STATIONS USING DYNAMIC PROGRAMMING

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CERTIFICATE

This is to certify that this work has been carried out under my supervision and it has not been submitted else where for a degree.

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L.P. Singh

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CHAPTER - I

INTRODUCTION

The economic load scheduling of a power system is perhaps the most exciting branch for the power system engineers. Ofcourse this was not very important in the beginning when there were small power generating stations for each locality such as urban power system etc. But now with the growth in the demand of electricity and at the same time assurance regarding the continuity of supply to the consumers under normal conditions, have forced the power system engineers to develop grid system i.e., interconnection of different generating stations located at different places. For such system, the economic load scheduling of the different generating plants in the system has become increasingly important.

By economic load scheduling we mean to find the generations of the different plants so that the total fuel cost is minimum, and at the same time the total demand and the losses at any instant must be met by the total generation. In the urban systems the generators are close to the load i.e. length of the transmission line is so small that the transmission losses are negligible. At the same time earlier i.e. for the urban power systems, there used to be mostly thermal plants. The operating cost of these thermal plants is mainly cost of fuel. This cost function turns out to be a nonlinear function of plant generations.

Normally the graph is given between the Heat value of fuel in B.Th.U. and power generation in M.W. and knowing the cost of the fuel, we can determine the fuel cost as a function of generation for each thermal plant.

Hence for the optimal operation the problem becomes to find the generation of the respective thermal plants i.e. P_i, $i=1,\ldots,N$ such that the objective function (i.e. total cost of fuel) as defined by the equation :

$$C_t = \sum_{i=1}^{N} C_i(P_i) \tag{1.1}$$

is minimum, subject to the constraint

$$\sum_{i=1}^{N} P_i = P_L \tag{1.2}$$

where P_L = total demand

and P_i = generation of the ith thermal plant.

This means the total demand at any instant is met by the total generation as the transmission losses for these schemes are negligible. This is only true for urban power systems.

The nonlinear objective function as given by the Eqn.(1.1) subject to the equality constrant (Eqn.1.2) may be converted by choosing Lagrange multiplier λ , into an unconstrained objective function as shown below

$$F = C_t - \lambda\left(\sum_{i=1}^{N} P_i - P_L \right) \tag{1.3}$$

This can be solved for minimum by simple law of calculus i.e. by determining the partial derivative of the function 'F' with respect to variable P_i (plant generation) and equating it equal to zero.

i.e. $$\frac{\partial F}{\partial P_i} = 0 = \frac{\partial C_i}{\partial P_i} - \lambda$$

or $$\lambda = \frac{\partial C_i}{\partial P_i} = \frac{dC_i}{dP_i} \qquad (1.4)$$

Hence for the most economic operation all plants must operate at the same incremental cost. The incremental cost is normally given in B.Th.U./M.W. and knowing the cost of fuel, it can be converted into actual cost function.

However with the development of integrated power systems (i.e. grid systems) and also with the interconnections of different power station located at far places for the purpose of economy interchange and reliability of operation, it is necessary to consider not only the incremental fuel cost but also the incremental transmission losses for economy. This is because no doubt the operating cost i.e. the generation cost of any thermal plant may be low, but still the plant may be uneconomical because of it being located very far from the load centre. Due to this the transmission losses may be considerably high and hence the plant may be overall uneconomic. A transmission loss formula expressing the total transmission losses as function of plant generation was first presented by George[1] in 1943. Since then there has been considerable

improvement in the method for the determination of the transmission losses as a function of plant generation. The application of digital computer to calculate losses was developed by Kirchmayer, Stagg, Glimn and Habermann[2]. The form of the loss formula is given below :

$$P_T = \sum_{m=1}^{N} \sum_{n=1}^{N} P_m B_{mn} P_n + \sum_{n=1}^{N} B_{no} P_n + B_{oo} \qquad (1.5)$$

where P_m, P_n = Plant generations = $(P_1 \; . \; . \; . \; . \; P_N)$

and B_{mn}, B_{no}, B_{oo} = Loss coefficient.

Therefore the optimal load scheduling for thermal plants taking transmission losses into consideration will be mathematically formulated as shown below.

Determine the plant generation P_i, $i = 1, \ldots, N$ such that the cost function

$$C_t = \sum_{i=1}^{N} C_i(P_i)$$

is a minimum subject to the constraint that the demand and the transmission losses at any instant must be met by the total generation at that instant

i.e. $$\sum_{i=1}^{N} P_i = P_L + P_T$$

where P_T is transmission loss which is a function of plant generation P_i.

As already shown for the no loss case, the above nonlinear objective function with nonlinear equality

constraint, can be converted into unconstrained objective function by choosing a proper value of Lagrange multiplier as shown below :

$$F = C_t - \lambda\left(\sum_{i=1}^{N} P_i - P_L - P_T \right) \qquad (1.6)$$

Proceeding as before, we get

$$\frac{\partial F}{\partial P_i} = 0 = \frac{dC_i}{dP_i} - \lambda\left(1 - \frac{\partial P_T}{\partial P_i}\right)$$

i.e. $$\lambda = \frac{dC_i}{dP_i} + \lambda\frac{\partial P_T}{\partial P_i}$$

or $$\lambda = \frac{dC_i}{dP_i} \Big/ \left(1 - \frac{\partial P_T}{\partial P_i}\right) \qquad (1.7)$$

where $\dfrac{1}{1-\dfrac{\partial P_T}{\partial P_i}} = \alpha$ = Penalty factor of plant i which depends upon the location of the plant. Hence the effect of the transmission losses is to add penalty factor in the solution where the value of penalty factor depends upon the plant location.

(Uptilnow we have discussed only the thermal system.) In the next chapter ~~a~~ A brief review is given for the economic load scheduling of the hydro thermal and thermal station. Subsequently the optimisation of the combined hydro thermal stations including the effect of transmission losses, has been studied. The problem is formulated by the use of dynamic programming and is tested for three thermal and one hydel plant. Actually the optimum load scheduling of the hydro thermal plants including the effects of transmission losses by Dynamic

Programming requires further investigation as this aspect has been neglected earlier in the problem formulation. However these aspects including the problem formulation and its solution etc. will be discussed in the subsequent chapters.

CHAPTER - II

ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL POWER STATIONS

(Brief Review)

In the initial stage, there were mostly thermal plants to produce electricity but due to the increase in the demand of electricity for all purposes together with high cost of fuel as well as its limited reserve, has focussed considerable attention upon the need for water power development. As the operating cost of thermal plant is very high and at the same time its capital cost is low compared to the hydro-electric plant whose capital cost is high but operating cost is low, it has become economical as well convenient to have both thermal as well as hydro plants in the same grid. The thermal plant is run as a base load plant and the hydro-electric plant is run as a peak load plant. This is because the hydro-plant can be started quickly, has higher reliability and greater speed of response and hence it can take up fluctuating loads. In other words the hydel plant is worked as a peak load plant to take care of the fluctuating load and thermal plant whose speed of response and starting is slow, is run as the base load plant. In any case there is an advantage of having both (a) thermal plants with low capital cost and high operating cost and (b) the hydro-plant having high capital cost and low operating cost, in the same interconnected system so as to

attain reliability of supply to the various consumers. These plants can be installed at the suitable sites depending upon the economy and convenience in operation of respective plants.

This work is based upon the short range hydro-thermal economic optimization problem formulated earlier by Chandler, Dandeno, Glimm and Kirchmayer[3] (CDGK). The main cost of operation of the hydro-thermal plants is the cost of fuel used in the thermal plants as the cost of water for the hydel plant is negligible compared to the cost of fuel of the thermal plant. Hence the problem is to find the generation of individual plants, both thermal as well as hydel, such that the total fuel cost is minimum and at the same time total demand P_L and losses P_T is continuously met. As it is a short range problem there will not be any appreciable change in the level of water in the reservoirs and hence head of water in the reservoir of the hydro plants will be assumed to be constant during the interval. However specified quantity of water Y_i must be utilised within the interval at each hydro plant i. The problem is formulated as follows :

Find the thermal generation S^j and the hydro generation h^i which are function of time, such that the cost functional as defined by

$$C_T = \int_0^T \sum_{j=1}^{N} C_j(S_j)\, dt \qquad (2.1)$$

is minimum, subject to the equality constraint

$$\sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_L + P_T\,(S,H) \qquad (2.2)$$

i.e. the total demand P_L and the total losses P_T at any instant is equal to the total generation of the plants, and

$$\int_0^T y_i(h_i)\, dt = Y_i \qquad (2.3)$$

i.e. in the given interval T, specified quantity of water Y_i is utilised at the hydel plant i, and the different symbols are as shown below :

S_j = Power generation of the jth thermal plant

h_i = Power generation of the ith hydel plant

T = Final time defining the short range problem

$C_j(S_j)$ = Fuel cost per hour of the jth thermal plant which is the function of the plant generation S_j

$y_i(h_i)$ = Turbine discharge in cubic ft./hour of the ith hydel plant which is the function of h_i

S = $(S_1 \; . \; . \; . \; . \; . \; S_N)$ = Generation of N thermal plants

h = $(h_1 \; . \; . \; . \; . \; . \; h_M)$ = Generation of M hydel plants

P_L = Demand at any time t

P_T = Transmission losses which are function of plant generations.

The above is the nonlinear programming problem with both the objective function (2.1) as well as the constraint equations (2.2, 2.3) being nonlinear. As discussed earlier this nonlinear objective function with the nonlinear equality constraints can be converted to the unconstrained objective function with the proper choice of the Lagrange multipliers λ and γ. Hence proceeding as earlier, the following coordination equations are obtained[3].

$$\frac{dC_j}{dS_j} + \lambda \frac{\partial P_T}{\partial S_j} = \lambda \qquad (2.4)$$

$$\gamma_i \frac{dy_i}{dh_i} + \lambda \frac{\partial P_T}{\partial h_i} = \lambda \qquad (2.5)$$

for $j = 1,\ldots,N$ and $i = 1,\ldots,M$.

The Lagrange multiplier γ_i is constant during the interval. The direct application of the above coordination equations give solution which some times dictates generations outside the plant capacity and also negative generation of certain plants because restrictions imposed by such capacity limitations have not been included in the problem formulation. Dandeno[4] also observed similar phenomena while working with these coordination equations on the digital computer.

Therefore the problem is extended here by considering the situation that exists in practice. This extension consists of the addition of simple bounds on the operating range of each plant as indicated by the inequalities (2.6) and (2.7).

$$\underline{S}_j \leqslant S_j \leqslant \overline{S}_j \qquad (2.6)$$

and $$\underline{h}_i \leqslant h_i \leqslant \overline{h}_i \qquad (2.7)$$

where $\underline{S}_j$ and $\underline{h}_i$ are the minimum and $\overline{S}_j$ and $\overline{h}_i$ are the maximum limits of operation of the corresponding plant. The preliminary step in the extension is the application of Pontryagin's Principle. By applying this Maximum Principle followed by the Kuhn Tucker conditions, the problem obtained is to maximize the Hamiltonian H at each instant of time (Wije Perera[5]).

i.e. Maximize $H = - \sum_{j=1}^{N} C_j(S_j) - \sum_{i=1}^{M} \gamma_i y_i(h_i)$ (2.8)

The maximization of H at each instant of time is an auxiliary problem of static type. The constraint γ_i must be so chosen that the boundary conditions given by equation (2.3) is satisfied, i.e. specified quantity of water is used at each hydro-plant in the specified period. Since γ_i effectively converts the water consumption in cubic ft/hr. to the cost, it may be termed as water value. Thus γ_i actually corresponds to the price of water and an increase in the value of γ_i results in lesser water usage at the ith hydel plant and vice-versa.

Now as the minimization of any function is equivalent to the negative of the maximization of the same function, and hence the maximisation of the Hamiltonian 'H' at each instant of time is equivalent to the minimization of the function C_T as shown below :

Minimize $C_T = - H$

$$= - \left(- \sum_{j=1}^{N} C_j(S_j) - \sum_{i=1}^{M} \gamma_i y_i(h_i)\right)$$

$$= \sum_{j=1}^{N} C_j(S_j) + \sum_{i=1}^{M} \gamma_i y_i(h_i) \qquad (2.9)$$

The minimization of the function C_T as defined by the equation (2.9) corresponds to the minimization of the total operating cost of the thermal as well as the hydel

plants. This is because both the terms in cost function C_T (2.9) represent the operating cost of the combined hydro-thermal system, the first term representing operating cost of the thermal plant and the second term representing the operating cost of the hydel plant.

CHAPTER - III

DYNAMIC PROGRAMMING

(Introduction and Formulation)

Dynamic Programming was developed in 1950 by R.Bellman[6] and his associates. This method was developed to study the optimization arising in the industry, economics, defence and in the social services where the modern optimization technique of linear and nonlinear programming and calculus of variation and its generalization are not applicable. Such catagories of problems are referred to multi-decision process, where we are required to take a sequence of decision at each stage.

Let us consider a system which can be described at discrete times by a finite number of variables, $X_1, X_2, \ldots, X_n$ which are referred to as state variables.

Let the value of these variables at stage i (i.e. at the instant of time i) be denoted by $X_1(i), X_2(i), \ldots, X_n(i)$. Hence we can define a state vector 'X' such that

$$X = X_1, X_2, \ldots, X_n$$

and its value at the stage i will be given by

$$X(i) = X_1(i), X_2(i), \ldots, X_n(i)$$

Suppose at each stage i, we have to take a decision $d(i)$ from amongst number of possible choices, which may be finite or infinite. Actually the effect of taking decision $d(i)$ at the stage i is to change the stage of the system $X(i)$ to $X(i+1)$. This can be stated mathematically as

$$X(i+1) = G\left[X(i), d(i), i\right]$$

where G is a functional. The above equation indicates that the state of the system at the stage $i+1$ depends upon the state of the system at the stage i i.e. $X(i)$, decision taken at the stage i i.e. $d(i)$, and the stage i itself.

Now because of taking a decision $d(i)$ at the stage i, we also get some return function $R(i)$ as shown below :

$$R(i) = R\,(X(i),\ d(i),\ i)$$

and hence the return function R depends upon $X(i)$, $d(i)$ and i.

We can also impose some constraint at the stage i such as

$$\psi\,(x(i),\ d(i),\ i) \leqslant \quad \geqslant = 0$$

If the total number of stages are finite say N, then at each stage we have the corresponding decisions $d(1)$, $d(2)$, ..., $d(N)$ and the total return 'I' associated with N-decision process will be

$$\begin{aligned} I &= I(R(1),\ \ldots,\ R(N)) \\ &= \sum_{i=1}^{N} R(i) \end{aligned}$$

where $R(1)$, $R(2)$ etc. are the return at the stage 1,2,etc.

Therefore our problem becomes to take decision $d(1)$, $d(2)$, .. etc. at the stages 1,2,... such that the total return function 'I', because of taking these decisions, is optimal.

The above system is known as Discrete time multistage decision process. In contrast to this there can be a process where we are required to take a continuum of decisions. Such processes are referred to as continuous time decision process. The process however, can be called, finite or infinite stage according to whether the number of stages 'N' is finite or infinite. Similarly if the function G,R, ψ defining such a system do not involve chance elements, the system is referred to as deterministic, otherwise it is known as probabilistic system. If the probability distribution of all the random variables occuring in the probabilistic distribution process are known, the system is known as stochastic decision process. Moreover the process is called stationary if each of the function G,R, etc. are independent of stage 'i'.

Mathematical Formulation:

As we are going to use discrete time, deterministic, stationary, finite multistage decision process in the formulation of the generation scheduling problem, only the mathematical formulation of such system will be dealt with. Our problem is to determine the optimal value of 'I' and the corresponding optimal sequence of decisions, one decision at each stage.

Actually technique of dynamic programming is based upon the principle of optimality by Bellman which states that whatever be the initial state and initial decisions, the remaining decisions must constitute an optimal sequence of decisions with regards to the state resulting from the first decision.

Now it is obvious that the value of 'I' depends upon N, the total number of stages to be considered and the initial state of the system say C. In view of this let

$F_N(C) =$ Optimal value of I when N-stages remain and the initial stage is C.

If the first decision d(1) is taken arbitrarily, then corresponding to this decision d(1), we have

$$X(2) = G\ (C,\ d(1))$$

Hence the maximum return over the remaining N-1 stages, the initial state being X(2), is

$$F_{N-1}\ (G\ (C,\ d(1)))$$

and the return from the first stage, when decision d(1) is taken, is

$$R\ \Big[C,\ d(1)\Big]$$

Therefore when the first decision, d(1), is arbitrary, and the remaining N-1 decisions are optimal with regards to the state resulting from decision d(1), the total return is

$$R\ \Big[C,\ d(1)\Big]\ +\ F_{N-1}\Big[\ G\ \big[C,\ d(1)\big]\Big]$$

Thus using the principal of optimality, we must have

$$F_N(C) = \underset{d(1)}{\text{Maximum permissible}}\ R\Big[\big[C, d(1)\big] + F_{N-1}\big[G\ C,\ d(1)\big]\Big]$$

$$= R\ \Big[C, \big[d_N(C)\big]\Big]\ +\ F_{N-1}\Big[\ G\ \big[C,\ d_N(C)\big]\Big] \qquad (3.1)$$

so that $d_N(C)$ is the first optimal decision to be taken when the system starts with state C and N stages remain.

Further when N=1, we get from the above equation

$$F_1(C) = \underset{d(1)}{\text{Maximum permissible}} \left[R\left[C,\ d(1)\right]\right]$$

$$= R\left[C,\ d_1(C)\right] \qquad (3.2)$$

Similarly by putting $N = 2$, we get

$$F_2(C) = \underset{d(1)}{\text{Maximum permissible}} \left[R\left[C, d(1)\right] + F_1\left[G\left[C, d(1)\right]\right]\right]$$

$$= R\left[C,\ d_2(C)\right] + F_1\left[\ G\left[C, d_2(C)\right]\right] \qquad (3.3)$$

In this way we can compute $F_1(C)$ from equation (3.2) for any possible initial state C and similarly equation (3.3) can be used to compute $F_2(C)$ and so on. Continuing like this we can compute $F_N(c)$ and $d_N(c)$ for any value of N and any possible initial state C. Thus $F_N(C)$ gives us the maximum value of I for the N-stage process when the initial state is C. Thus we observe that at any iteration, we have to solve the minimization problem in one variable only. This is the main reason of popularity of the dynamic programming. The main advantage is, instead of meeting n difficulties simultaneously, try to meet these one at a time.

CHAPTER - IV

FORMULATION OF LOAD SCHEDULING PROBLEM OF HYDRO-THERMAL STATIONS USING DYNAMIC PROGRAMMING

Let us assume that the interconnected system has both the thermal as well as the hydel plants. Let S_j be the generation of the jth thermal plant and let h_i be the generation of the ith hydel plant. And let us assume that our system contains N thermal stations and M hydel stations. It will be assumed that the thermal plants are run as a base load plant and therefore we shall assume that first of all, depending upon the demand, all the N thermal plants will be put into operation one by one and afterwards when the demand exceeds the sum of the lower bound of all the thermal plants, hydel plant may be put into operation. This is because the hydel plants are run as the peak load plant and therefore they supply the fluctuating load. The hydel plants will also be put into operation one by one until all the M hydel plants are exhausted. However the maximum demand that can be handled will be the sum of the upper bounds of all the N thermal and M hydel plants.

Therefore our problem is to determine the scheduling of generation of the hydro-thermal plants i.e. to determine S_j and h_i, such that the total operating cost is optimal i.e. the function C_T as defined below is minimum.

$$C_T = \sum_{j=1}^{N} C_j(S_j) + \sum_{i=1}^{M} \Gamma_i y_i(h_i) \qquad (4.1)$$

where C_T is the operating cost of the hydro-thermal system

and $\Gamma_i y_i(h_i)$ is the cost characteristics of the ith hydel plant.

However if we do not put any constraint in the minimization of the objective function as defined by the equation (4.1), we will get the trivial solution of zero cost with zero generation. Therefore the following constraint must also be satisfied.

$$\sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_L + P_T(S, h) \qquad (4.2)$$

Care should also be taken that the solution obtained should not dictate generation of any plant beyond the plant capacity or negative generation. Hence the following constraints must also be satisfied.

$$\underline{S}_j \leqslant S_j \leqslant \overline{S}_j \qquad (4.3)$$

and $$\underline{h}_i \leqslant h_i \leqslant \overline{h}_i \qquad (4.4)$$

However the above constraints are only effective if the corresponding plants are in operation. Because if any plant is not in operation i.e. if any plant is shut down, its operating cost will be zero i.e.

$$C_j(o) = 0 \quad \text{and} \quad \Gamma_i y_i(o) = 0 \qquad (4.5)$$

In order to use Dynamic Programming as described earlier, we have to define a function which should correspond to the minimization of the cost function C_T (4.1) and should also take into consideration the constrained equation (4.2). In other words, our problem is to determine the optimal decision from amongst multidecisions i.e. the optimum generation S_j and h_i out of multidecisions (i.e. out of several generations) such that the total return function (i.e. the cost function C_T defined by the equation 4.1) because of taking the decisions S_j and h_i, is optimum subject to the constraints defined by the equations (4.2), (4.3) and (4.4). It is obvious that the optimal (i.e. minimum) value of C_T depends upon the total number of stages to be considered which corresponds to the total number of plants (M + N plants in this case) and the initial state of the system which corresponds to the initial demand 'D'. As a matter of fact the initial state of the system takes into account the constraint equations (4.2), (4.3) and (4.4). The initial demand 'D' will thus be equal to $P_L + P_T$. Let this function be $F_R(D)$.

Hence $F_R(D)$ = Minimum value of C_T, when there are R stages i.e. there are R plants in operation and the initial state is D which corresponds to the demand at that instant. These R plants may be thermal or both thermal and hydel such that the demand D satisfies the constraints similar to the equations (4.2), (4.3) and (4.4).

The following constraints should also be satisfied by the demand D :

$$\underline{D} \leqslant D \leqslant \overline{D} \qquad (4.6)$$

Here $\underline{D}$ and $\overline{D}$ are the lower and upper bounds of D respectively. The lower bound of D i.e. $\underline{D}$ is the sum of the lower bound of all the plants which are in operation during that instant and the upper bound of D i.e. $\overline{D}$ is the sum of the upper bounds of all the plants which are in operation. Now if any plant is shut down, its operating cost is zero.

Let us assume that out of total number of R plants which takes care of the total demand D on the plants, the Rth plant supplies 'Z demand'. That is the generation of the Rth plant is Z because the demand at any plant is actually equal to the generation of that plant. Hence by taking this first decision i.e. by taking the decision that the demand at the Rth plant is Z, the remaining R-1 plants will have to supply the remaining demand D-Z.

Let the cost of generating Z by the Rth plant be $C_R(Z)$ if this plant is thermal otherwise $\gamma_R\, y_R(Z)$ if the Rth plant is hydel. Let this cost be represented by $U_R(Z)$ with the restriction that

$U_R(Z) = C_R(Z)$ if the Rth plant is thermal and

$U_R(Z) = \gamma_R\, y_R(Z)$ if the Rth plant is hydel.

Then the optimum cost of generating the remaining demand of D-Z by the remaining number of plants R-1 will also be defined by the similar function as described earlier. Therefore we have

$F_{R-1}(D-Z)$ = Minimum cost of generating D-Z demand when there are R-1 plants in operation. The

demand of (D-Z) must also lie between the sum of the lower bounds and the sum of the upper bounds of these R-1 plants which are in operation at this instant. Therefore we have the following equation by using the principle of optimality.

$$F_R(D) = \underset{Z}{\text{Min}} \left(U_R(Z) + F_{R-1}(D-Z)\right) \qquad (4.7)$$

where $U_R(Z)$ corresponds to the initial decision that the Rth plant takes the demand of Z which is arbitrary and $F_{R-1}(D-Z)$ corresponds to the remaining R-1 decisions which are always optimal with regards to the state resulting from the first i.e. the initial decision. Actually by taking the first decision that the Rth plant takes a demand of Z, the state of the system is changed to D-Z.

By putting R = 1 in equation (4.7) we obtain

$$F_1(D) = \underset{Z}{\text{Min}}\ U_1(Z)$$

$$= U_1(D) \qquad (4.8)$$

= Minimum cost of generating 'D' demand when only the first plant is in operation.

According to our formulation the first plant to be put into operation will be the thermal plant which has the cost characteristics of $C_1(S_1)$ and hence we have

$$F_1(D) = U_1(D)$$

$$= C_1(D) \qquad (4.9)$$

= Minimum cost of generating the demand D by the first thermal plant.

The demand D must satisfy the constraint equation (4.6).

i.e.

$$\underline{S}_1 \leqslant D \leqslant \overline{S}_1$$

where $\underline{S}_1$ and $\overline{S}_1$ is the lower and upper bounds of the first thermal plant. Similarly putting $R = 2$, i.e. when there are two plants in operation, we obtain from the equation (4.7)

$$F_2(D) = \underset{Z}{\mathrm{Min}} \left(U_2(Z) + F_1(D-Z)\right) \qquad (4.10)$$

As the 2nd plant is also thermal, we have from the above

$$F_2(D) = \underset{Z}{\mathrm{Min}} \left(C_2(Z) + F_1(D-Z)\right)$$

$$= \underset{Z}{\mathrm{Min}} \left(C_2(Z) + C_1(D-Z)\right)$$

Here Z varies between the limits

$$\underline{S}_2 \leqslant Z \leqslant \overline{S}_2$$

where $\underline{S}_2$ and $\overline{S}_2$ are the lower and upper bounds of the second thermal plant and D-Z will vary between the limits

$$\underline{S}_1 \leqslant (D-Z) \leqslant \overline{S}_1$$

Similarly we can go on finding $F_3(D)$, $F_4(D)$ .... till we obtain $F_N(D)$ i.e. when all the N thermal plants are put into operation.

Due to our basic assumption when the first hydel plant is put into operation, all the N thermal plants must be operating we draw the conclusion that when N+1 plants are in operation

naturally N plants are thermal and one plant is hydel. The hydel plants will also be put into operation in the same sequence, one by one, until all the N thermal plants and M hydel plants are put into operation.

Hence putting R = N+1 in the equation (4.7) we obtain

$$F_{N+1}(D) = \min_{\underline{h}_1 \leqslant Z \leqslant \overline{h}_1} (\gamma_1 y_1(h_1) + C_N(D-Z))$$

$$= \min_{\underline{h}_1 \leqslant Z \leqslant \overline{h}_1} (\gamma_1 y_1(Z) + C_N(D-Z)) \qquad (4.11)$$

where $\underline{h}_1$ and $\overline{h}_1$ are the lower and upper bounds of the first hydel plant.

The above is true because (N+1)st plant is actually the first hydel plant. The process is continued till we obtain $F_{N+M}(D)$. In other words the iteration is continued till all the available N+M plants are put into operation and the demand D given by the constraint equation (4.2) is satisfied. The maximum value of D is restricted by the following equation :

$$D \leqslant \left(\sum_{j=1}^{N} \overline{x}_j + \sum_{i=1}^{M} \overline{h}_i \right) \qquad (4.12)$$

We observe here that at every iteration of the process described, we have to solve a minimisation problem with respect to one variable Z (scalar) only, which can be carried using a suitable step size. In the process we also obtain the optimal plant generation. However for the sake of convenience both types of plants i.e. thermal and hydel are numbered in serial

order because in this method we can determine not only the minimum cost to produce the demand D but also optimal loading or generation of the individual plants.

The short range problem of 24 hours duration has been considered and divided into interval of one hour each. The demand D at the generating plant is assumed to be constant during each hourly interval and at the end of each interval, the demand increases or decreases in jumps or it can remain the same as before. That is to say when only thermal plant 1 is in operation, we have

$$\underline{S}_1 \leqslant D \leqslant \overline{S}_1$$

such that D varies between $\underline{S}_1$ and $\overline{S}_1$ in steps as shown below :

$$\underline{S}_1 \ , \ \underline{S}_1 + x \ , \ \underline{S}_1 + 2x \ , \ \ldots\ldots\ldots\ldots, \ \overline{S}_1$$

where x can be any integer such as 1,2,....etc.

Further in the case of hydel plant we assign some initial cost of water i.e. we assign some price to the Lagrange multiplier γ_i. Now if a higher cost is attached to γ_i i.e. if we start with greater price of water, naturally less water is required to generate h_i at the ith hydel plant. But this must satisfy the constraint as shown below :

$$\int_t^o y_i(h_i) = Y_i \qquad (4.13)$$

Hence if the above constrained equation is not satisfied, we have to refix the value of γ_i. As it is a short range problem, the level of water (i.e. head of water) in the reservoir will remain constant and the effect of evaporation or rainfall etc.will be negligible

CHAPTER - V

SOLUTION OF A HYDRO-THERMAL SYSTEM

The economic load scheduling of the Hydro-thermal system that we are going to study is predominantly a thermal system with one or two Hydel plants. This is because of our basic assumption that the thermal plants are operated as base load plants and the hydel plants are operated as peak load plants. However in case the number of hydel plants are more than one, they can always be combined into a single equivalent plant on the assumption that each of the hydro-stations are located at about the same distance from the load centre. This means that the incremental transmission loss is about the same. Thus after obtaining the desired solution i.e. the generation scheduling of the equivalent hydro-plant, we can assign this generation (i.e. power production) to the various hydro plants in proportion to their capacities.

Economic load scheduling using dynamic programming has been developed initially for three thermal and one equivalent hydro-plant. In the initial stage of problem formulation, it has been assumed that the total demand D at the plants are known. Thus after the determination of the optimum cost and the corresponding generation schedule of the different thermal and hydel plants of the system, the transmission losses are calculated using loss coefficient (Appendix B). Then with the help of the following equation, load at different intervals is calculated.

$$P_L + P_T(S, h) = D \qquad (5.1)$$

We have taken the following numerical example for our system. The system consists of three thermal plants and one hydel plant. The cost characteristics of the different thermal plants and discharge characteristics of the hydel plants as a function of the plant generations are known. These are usually non-linear characteristics of the plant generation as shown

$$C(S) = a_0 + a_1 S + a_2 S^2$$

where $C(S)$ = cost function

S = generation

and a_0, a_1, a_2 = constants

We are also given the upper and lower bounds of the respective plants and total quantity of water to be utilized by the equivalent hydel plant in the specified interval. We are also given the demand 'D' at the plants during different hours. The following example is studied.

a) Thermal Plant

S. No.	Lower bound	Upper bound	Cost Characteristic	Quantity of water to be utilized	
1	50	150	$C_1 = 100 + 0.1S_1 + 0.01S_1^2$	-	Tens of Rs./hr.
2	60	150	$C_2 = 120 + 0.1S_2 + 0.02S_2^2$	-	
3	50	200	$C_3 = 150 + 0.2S_3 + 0.01S_3^2$	-	
			b) Hydel Plant		
4	15	65	$y_1 = 140 + 20h_1 + 0.06h_1^2$	18,000	Cu.ft.

The demand D, for the short range problem of 24 hours duration which has been considered here, is shown by Figure 1. The demand, as shown in the figure, remains constant during each hourly interval and then at the end of the interval, it increases or decreases in jumps or remains constant.

Thus the problem is to determine thermal generations S_1, S_2 and S_3 and the hydel generation h_1 during each interval of time such that the cost function

$$C_T = \sum_{j=1}^{3} C_j(S_j) + \gamma_1 y_1(h_1) \qquad (5.2)$$

is minimum, subject to the constraint

$$\sum_{j=1}^{3} S_j + h_1 = P_L + P_T = D \qquad (5.3)$$

and $$\int_0^T y_1(h_1)\, dt = Y_1 \qquad (5.4)$$

Thus putting R = 1 in the equation (4.7), we obtain the following function for the first thermal plant

$$F_1(D) = \min_Z (C_1(Z)) \qquad (5.5)$$

Here Z is varied between $\underline{S}_1$ and $\overline{S}_1$ in discrete steps and for every Z, we determing the corresponding cost $F_1(D)$. Here in our problem we have varied Z in the step of 5 M.W. Thus we obtain

$$F_1(D) = C_1(D) \qquad (5.6)$$

$$\text{for } \underline{S}_1 \leqslant D \leqslant \overline{S}_1$$

where D is varied between the above limits in the step of 5 M.W.

Again putting R = 2 in the equation (4.7), we obtain the optimal cost and the corresponding generation schedule when both the thermal plants 1 and 2 are operating. Hence we obtain

$$F_2(D) = \min_{\underline{Z}_2 \leqslant Z \leqslant \overline{Z}_2} (C_2(Z) + F_1(D-Z)) \qquad (5.7)$$

Here the demand D is varied between the limits

$$\underline{D} \leqslant D \leqslant \overline{D} \qquad (5.8)$$

where $\underline{D}$ and $\overline{D}$ are the sum of lower bounds and upper bounds respectively for the two thermal plants which are in operation at that instant. Thus for each D, we determine the optimal cost and the corresponding generation schedule of both the thermal plants i.e. generation schedule of the first thermal plant which is Z and the generation schedule for the second thermal plant which is D-Z. To obtain this, Z is varied between the above limits in the step of 5 M.W. and for each value of Z, the cost is calculated and compared with the previous one so as to obtain the optimal cost. However D-Z can only vary between the limits

$$\underline{Z}_1 \leqslant (D-Z) \leqslant \overline{Z}_1$$

The process is repeated for all values of D given by the equation (5.8). However $F_1(D-Z)$ is already known from the equation (5.6).

Similarly by putting R = 3, we obtain the following equation when three thermal plants are in operation.

$$F_3(D) = \min_{\underline{S}_3 \leq Z \leq \overline{S}_3} (C_3(Z) + F_2(D-Z))$$

$$= \min_{\underline{S}_3 \leq Z \leq \overline{S}_3} (C_3(Z) + C_2(D-Z)) \qquad (5.9)$$

Here again D is varied between the sum of the lower bounds and the sum of the upper bounds of the three thermal plants and for every D, the optimal cost to generate this demand and the corresponding generation schedule is determined. This is done in the same way as discussed earlier by varying Z between the limits as given in the equation (5.9), calculating the corresponding cost and comparing them in order to determine the optimal cost. The corresponding generation schedule for the third thermal plant will be the value of Z which gives the minimum cost to produce the demand D. However $C_2(D-Z)$ and the corresponding (D-Z) are the generation schedule for the first two thermal plants which can be determined by the equation (5.7).

Now when the demand exceeds the sum of the lower bounds of all the thermal plants, the fourth plant which is the hydel plant may come into operation. Thus in the similar way we obtain the following :

$$F_4(D) = \min_{Z} (\gamma_1 y_1(Z) + F_3(D-Z))$$

$$= \min_{\underline{H}_1 \leq Z \leq \overline{H}_1} (\gamma_1 y_1(Z) + C_3(D-Z)) \qquad (5.10)$$

The demand D is given as shown in the Figure 1 for 24 hours. For each hourly demand 'D', the optimal cost of generation and the generation schedule of the respective plants is determined in the way explained earlier. This is done by initialising the value of γ_1. The value of γ_1 depends upon the quantity of water to be used. We assign a low value for γ_1 to start with. Thus after determining the generation schedule h_1 for the hydel plant during the entire interval of 24 hours, the total discharge in the period is calculated from the given discharge characteristics equation of the hydel plant. The value of total discharge thus obtained is compared with the specified quantity of water to be utilised by this hydel plant during the interval

i.e. $$\int_0^T \gamma_1 y_1(h_1)\, dt = Y_1 \qquad (5.11)$$

Initially if we assign a low value to γ_1 we will most likely obtain a higher value of water consumption than specified. Thus the value of γ_1 is incremented by $d\,\gamma_1$ and again the optimal cost and the corresponding generation schedule is obtained. The process is repeated until the equation (5.11) is satisfied. However D-Z in the equation (5.10) which is the generation schedule for all the three thermal plants is divided optimally between the three thermal plants by the equation (5.9), so that the entire system may operate with optimum cost. Initially a value of γ_1 was chosen = 0.1 and $d\,\gamma_1$ was taken as 0.01.

The optimum cost to generate the demand D and the corresponding generation schedule is given in the tabular form on page 33. The transmission losses have also been calculated (Appendix B) for the given system for the values of generation schedule thus obtained. This is done by taking the assumed value of loss coefficients (Table 1). Hence substracting the values of transmission losses P_T thus obtained, from the Demand D, the actual load to be supplied from this sytem is also tabulated on the same page. The flow diagram and the computer programme printout are given in Appendix A.

Table 1

Loss Coefficient

0.0005	0.00005	0.0002	0.00003
	0.00004	0.00018	-0.00011
		0.0005	-0.00012
			0.00023

RESULTS

Hours	Demand D	Optimal Cost	GENERATION SCHEDULE Thermal Plant P1			Hydel Plant h_1	Trans. Losses P_T	Load Supplied P_L
			P_1	P_2	P_3			
1	175	585.55	100	60	00	15	5.68	169.32
2	190	603.80	115	60	00	15	7.40	182.60
3	220	647.30	135	70	00	15	10.19	209.81
4	280	761.30	105	60	100	15	17.23	262.77
5	320	857.30	125	60	120	15	24.03	295.97
6	360	966.80	140	70	135	15	30.51	329.49
7	390	1057.59	150	75	150	15	36.29	353.71
8	410	1123.55	150	85	160	15	39.44	370.56
9	440	1230.64	150	90	175	25	43.23	396.77
10	475	1364.75	150	95	190	40	47.05	427.95
11	525	1573.95	150	110	200	65	50.16	474.54
12	550	1698.95	150	135	200	65	52.23	497.77
13	565	1785.95	150	150	200	65	53.49	511.51
14	540	1645.95	150	125	200	65	51.39	488.61
15	500	1466.39	150	100	200	50	49.87	450.13
16	450	1268.03	150	90	180	30	44.34	405.66
17	425	1176.23	150	85	170	20	41.74	383.26
18	400	1089.80	150	80	155	15	37.84	362.16
19	375	1011.30	145	75	140	15	32.95	342.05
20	340	910.55	135	65	125	15	27.15	312.85
21	300	807.30	115	60	110	15	20.49	279.51
22	250	699.80	150	85	00	15	12.72	237.28
23	200	617.30	125	60	00	15	8.67	191.33
24	180	591.30	105	60	00	15	6.25	173.77

CONCLUSION

Economic load scheduling of the Hydro-Thermal system essentially belongs to the category of problems known as multi-stage decision process which can be tackled more effectively by the Dynamic Programming. In the case of Economic load scheduling of the Hydro-Thermal system, we are required to take the sequence of decisions and at any decision point, number of possibilities exist just like multi-stage decision process. The decision that we are required to take in this case is the decision regarding the generation schedule of the respective plants. The stage corresponds to the number of plants that are in operation and the decision at any stage corresponds to the proper choice of the generation schedule out of several choices of generations. The effect of taking the decision at any stage will result into the change in the state of the system. Here the state corresponds to the demand. Thus the effect of decision regarding the generation schedule for any plant will naturally change the demand to be handled by the remaining plants. Just like multi-decision process, the state of the system at any stage depends upon the previous state, decision regarding generation schedule at that stage, and the stage i.e. plant itself. With this change in the state i.e. the demand on the system will naturally result into some return function which corresponds to the cost function for the hydro-thermal system. Therefore a sequence of decisions regarding the generation schedule are to be taken so that the total return function i.e. the total cost function because of

taking such decisions, are optimal.

Therefore we come to the conclusion that the economic load scheduling of the hydro-thermal system can be formulated and studied in a more effective way by dynamic programming because this problem also belongs to the categories of problems known as the multi-stage decision process as shown earlier.

If the system contains only thermal plants then the optimal cost to supply any demand and the corresponding generation schedule can easily be determined by the dynamic programming even the number of thermal plants are very large subject to the restrictions imposed by the computer memory. However if the system contains hydel plants as well, then the formulation through dynamic programming becomes difficult as the problem becomes dynamic in nature due to restrictions regarding water consumption by the hydel plant during the specified interval.. However the formulation through dynamic programming has beendeveloped here for a system which is predominantly thermal system with one or two hydel stations. In case the number of hydel plants are more than one, they can always be replaced by an equivalent hydel plant, which will have the total energy equal to the sum of the energy of the respective hydel plants. As a matter of fact the level of water i.e. head of water in the reservoir of the respective hydel plant corresponds to the energy which the hydel plant is capable of producing. But we can only combine the hydel plants to form an equivalent hydel plant if these plants are situated at about the

same distance from the load centre and thus they have the same incremental transmission losses. Therefore a further investigation is proposed if it is not possible to replace all the hydel plants by an equivalent hydel plant and thus the formulation for each hydel plant is to be considered separately.

In our formulation we have assumed the transmission losses initially as it is not possible to assess these losses correctly in the beginning as the plant loading upon which these losses depend are not known in the beginning. Thus for any given load and these assumed transmission losses, the demand on the plant is calculated. Afterwards for each demand, the optimal cost to produce this demand and the corresponding generation schedule i.e. plant loading using dynamic programming, has been calculated. The generation schedule thus obtained has been used to calculate these losses again for a given value of loss coefficients.

Investigation is thus proposed regarding the development of an iterative technique to compare the calculated losses with the losses assumed earlier and if they differ by an amount more than a chosen small number, the calculated losses should be substituted to recalculate the demand and hence the optimal cost and the corresponding generation schedule. The iteration may be continued till the above condition is satisfied i.e. the losses at this iteration to that of previous iteration are nearly equal. However this requires the test regarding the convergence of the formulation.

APPENDIX – A

FLOW DIAGRAM FOR HYDRO-THERMAL SYSTEM USING DYNAMIC PROGRAMMING

(Including Computer Printout)

The flow diagram for the economic load scheduling of the hydro-thermal system using dynamic programming is given here. The following symbols are used in the flow diagram.

NK = Total number of plants in the system

NS = Number of thermal plants in the system

NC = Number of coefficients in the cost/discharge function of the plant

C(J,I) = Jth coefficient of the Ith thermal plant in the cost function

y(J,I) = Jth coefficient of the Ith hydel plant in the discharge function

INZ = Increments in Z

LB(J) = Lower bound of the Jth plant

UB(J) = Upper bound of the Jth plant

Y(J) = Quantity of water to be utilised at the Jth hydel plant in the specified interval

NH = Number of hydel plant = 1

IDM = Demand on the plants during the specified interval of 24 hours

```
        *****************************************************************

$WATFOR        EEF048        L P SINGH
$IBFTC ELSDP  ****************************************
C       ECONOMIC LOAD SCHEDULING OF HYDRO THERMAL PLANT
C       USING DYNAMIC PROGRAMMING.
C
        REAL LB(50)
         DIMENSION OPCOST(100),OPC(100),UB(50),C(5,20),DUMMY(2,100),Y(10),
      1  IDM(25)
        COMMON W,NC,X,C
        READ(5,100)NK,NS,NC
C       NK=NUMBER OF PLANTS,NC=NO.OF COEFFICIENTS IN THE COST
C       FUNCTION OF THE PLANTS
C        NS= NUMBER OF THERMAL PLANTS
C
        DO1 I=1,NK
      1 READ(5,101)(C(J,I),J=1,NC)
C       C(J,I) IS THE JTH COEFFICIENT OF THE
C       ITH PLANT
        READ(5,102) DMZ
C       DMZ=INCREMENT OF Z AND D
        READ(5,103)(LB(J),UB(J),J=1,NK)
C       LB(J) AND UB(J) ARE THE LOWER AND UPPER BOUNDS
C       OF THE JTH PLANT
        NH=NK-NS
        READ(5,104)(Y(J),J=1,NH)
C        Y(J) = TOTAL QUANTITY OF WATER TO BE UTILYSED
C           AT THE J TH HYDEL PLANT
        READ(5,105)(IDM(J),J=1,24)
C       IDM IS THE LOAD DEMAND DURING 24 HOURS
C         ****************************************************
C
C
C       CALCULATION OF F1(D)
        D=LB(1)
        WRITE(6,205)
        WRITE(7,205)
        WRITE(6,200)
        WRITE(7,200)
        WRITE(6,205)
        WRITE(7,205)
        I=1
      2 OPCOST(I)=COST(D,1)
        WRITE(6,201)I,D,OPCOST(I)
        WRITE(7,201)I,D,OPCOST(I)
        WRITE(6,205)
```

```
      WRITE(7,205)
       DUMMY(2,I) = D
      I=I+1
      D=D+DMZ
      IF(D.LE.UB(1))GO TO 2
C       ***************************************
C     CALCULATION OF F2(D), F3(D).....
      DO3 I=2,NK
      WRITE(6,205)
      WRITE(7,205)
      WRITE(6,202) I
      WRITE(7,202) I
      WRITE(6,205)
      WRITE(7,205)
      WRITE(6,204)
      WRITE(7,204)
      WRITE(6,205)
      WRITE(7,205)
      LL=I-1
      ZM1=0.
      ZM2=0.
C     UPPER AND LOWER BOUNDS OF D AND Z
      DO4J=1,LL
      ZM1=ZM1+LB(J)
    4 ZM2=ZM2+UB(J)
      DMIN=ZM1+LB(I)
      DMAX=ZM2+UB(I)
C
C       **********************************************
      GAMMA=1.
C      GAMMA IS THE WATER VALUE (PRICE OF WATER)
      IF(I.LE.NS) GO TO 20
      IR=I-NS
C      CHOOSING THE PROPER VALUE OF GAMMA
      GAMMA=0.09
   21  GAMMA=GAMMA+0.01
   20 SUM=0.
      D=DMIN
      K=1
    5 ZMIN=D-ZM2
      ZMAX=D-ZM1
      IF(ZMIN.LT.LB(I))ZMIN=LB(I)
      IF(ZMAX.GT.UB(I))ZMAX=UB(I)
C
      Z=ZMIN
```

```
      DD=D-Z
      N=0
    6 N=N+1
      IF(DD.NE.DUMMY(2,N)) GO TO 6
      OX1=GAMMA*COST(Z,I)+OPCOST(N)
C
      OPZ = Z
    7 DD=D-Z
      N=0
    8 N=N+1
      IF(DD.NE.DUMMY(2,N)) GO TO 8
      VN=N
      OX2=GAMMA*COST(Z,I)+OPCOST(N)
      IF( OX2 .LT. OX1 ) OPZ = Z
      IF(OX2.LT.OX1)OX1=OX2
      Z=Z+DMZ
C
      IF(Z.LE.ZMAX) GO TO 7
C       CALCULATION OF DISCHARGE OF HYDAL PLANTIN THE
C     SPECIFIED INTERVAL
C
      SUM=SUM+COST(OPZ,I)
      OPC(K)=OX1
      WRITE(6,203)K,D,OPZ,OX1
      WRITE(7,203)K,D,OPZ,OX1
      DUMMY (1,K) = D
      IF(I.LT.NK)GOTO51
      D=IDM(K)
      GOTO52
   51 D=D+DMZ
   52 K=K+1
      IF(D.LE.DMAX) GO TO 5
      IF(I.LE.NS) GO TO 22
      WRITE(6,205)
      WRITE(7,205)
      WRITE(6,506)IR
      WRITE(7,506)IR
      WRITE(6,507)GAMMA
      WRITE(7,507)GAMMA
      WRITE(6,504)IR
      WRITE(7,504)IR
      WRITE(6,505)SUM
      WRITE(7,505)SUM
      WRITE(6,205)
      WRITE(7,205)
```

```
      IF(SUM.GT.Y(IR)) GO TO 21
   22 KG=K-1
      DO9IX=1,KG
       OPCOST(IX)=OPC(IX)
 9      DUMMY(2,IX) = DUMMY(1,IX)
    3 CONTINUE
C
C    ************************************************************
  100 FORMAT(3I4)
  101 FORMAT(5F8.2)
  102 FORMAT(F6.2)
  103 FORMAT(2F8.2)
  104 FORMAT(2F8.2)
  105  FORMAT(20I4)
C
C
C
  200 FORMAT(2X*OPTIMUM COST FOR PLANT ONE*/2X*I  DEMAND  OP.COST*)
  201 FORMAT(I4,2F8.2)
  202 FORMAT(2X*OPTIMUM COST FOR*I4,* PLANTS*)
  204 FORMAT(2X*I*,10X,*DEMAND*,10X,*Z*,10X,*OP.COST*)
 203   FORMAT(I3,3(4X,F12.6))
205   FORMAT(1X,100(1H-))
 506  FORMAT(2X,*VALUE OF GAMMA*,I4,*HYDEL PLANT*)
 507  FORMAT(2X,F12.4)
 504  FORMAT(2X,*QUANTITY OF WATER UTILIZED BY*,I4,*HYDEL PLANT*)
 505  FORMAT(2X,F12.2)
      STOP
      END          **********************************
$IBFTC COST
      FUNCTION COST (D,K)
      DIMENSION C(5,20)
      COMMON W,NC,X,C
C     C(J,I)=THE JTH COEFFICIENT OF THE ITH PLANT
C     NC=THE NO. OF COEFFICIENTS IN A PLANT COST FUNCTION
      COST=C(NC,K)
      NT=NC-1
      DO1 I=1,NT
      J=NC-I
    1 COST=COST*D + C(J,K)
      RETURN
      END          ********************************
$ENTRY
   4   3   3
  100.00      .10      .01
```

```
120.00      .10      .02
150.00      .20      .01
140.00    20.00      .06
5.00
 50.00   150.00
 60.00   150.00
 50.00   200.00
 15.00    65.00
18000.00
 180 190 200 220 250 280 300 320 340 360 375 390 400 410 425 440
 540 550 565 450  475  500  525
*************************************************************************
  OPTIMUM COST FOR PLANT ONE
  I  DEMAND  OP.COST
-------------------------------------------
    1    50.00   130.00

    2    55.00   135.75

    3    60.00   142.00

    4    65.00   148.75

    5    70.00   156.00

    6    75.00   163.75

    7    80.00   172.00

    8    85.00   180.75

    9    90.00   190.00

   10    95.00   199.75

   11   100.00   210.00

   12   105.00   220.75

   13   110.00   232.00

   14   115.00   243.75

   15   120.00   256.00
```

16	125.00	268.75
17	130.00	282.00
18	135.00	295.75
19	140.00	310.00
20	145.00	324.75
21	150.00	340.00

****

OPTIMUM COST FOR 2 PLANTS

I	DEMAND	Z	OP.COST
1	110.000000	60.000000	327.999996
2	115.000000	60.000000	333.749996
3	120.000000	60.000000	339.999996
4	125.000000	60.000000	346.749996
5	130.000000	60.000000	353.999996
6	135.000000	60.000000	361.749996
7	140.000000	60.000000	369.999996
8	145.000000	60.000000	378.749996
9	150.000000	60.000000	387.999996
10	155.000000	60.000000	397.749996
11	160.000000	60.000000	407.999996
12	165.000000	60.000000	418.749996
13	170.000000	60.000000	429.999996
14	175.000000	60.000000	441.749996
15	180.000000	60.000000	453.999992
16	185.000000	60.000000	466.749992
17	190.000000	65.000000	479.749992
18	195.000000	65.000000	492.999992
19	200.000000	65.000000	506.749992
20	205.000000	70.000000	520.749992
21	210.000000	70.000000	534.999992
22	215.000000	70.000000	549.749992
23	220.000000	75.000000	564.749992
24	225.000000	75.000000	579.999992
25	230.000000	80.000000	595.999992
26	235.000000	85.000000	612.999992
27	240.000000	90.000000	630.999992

28	245.000000	95.000000	649.999992
29	250.000000	100.000000	669.999992
30	255.000000	105.000000	690.999992
31	260.000000	110.000000	712.999992
32	265.000000	115.000000	735.999992
33	270.000000	120.000000	759.999992
34	275.000000	125.000000	784.999992
35	280.000000	130.000000	810.999992
36	285.000000	135.000000	837.999992
37	290.000000	140.000000	865.999984
38	295.000000	145.000000	894.999984
39	300.000000	150.000000	924.999984

****

OPTIMUM COST FOR 3 PLANTS

I	DEMAND	Z	OP.COST
1	160.000000	50.000000	512.999992
2	165.000000	50.000000	518.749992
3	170.000000	50.000000	524.999992
4	175.000000	55.000000	531.249992
5	180.000000	55.000000	537.999992
6	185.000000	60.000000	544.749992
7	190.000000	60.000000	551.999992
8	195.000000	65.000000	559.249992
9	200.000000	65.000000	566.999992
10	205.000000	70.000000	574.749992
11	210.000000	70.000000	582.999992
12	215.000000	75.000000	591.249992
13	220.000000	75.000000	599.999992
14	225.000000	80.000000	608.749992
15	230.000000	80.000000	617.999992
16	235.000000	85.000000	627.249992
17	240.000000	85.000000	636.999992
18	245.000000	90.000000	646.749992
19	250.000000	90.000000	656.999992
20	255.000000	95.000000	667.249992
21	260.000000	95.000000	677.999992
22	265.000000	100.000000	688.749992
23	270.000000	100.000000	699.999992
24	275.000000	105.000000	711.249992
25	280.000000	105.000000	722.999992
26	285.000000	110.000000	734.749992
27	290.000000	110.000000	746.999984
28	295.000000	115.000000	759.249984

29	300.000000	115.000000	771.999984
30	305.000000	120.000000	784.749984
31	310.000000	120.000000	797.749984
32	315.000000	120.000000	810.999984
33	320.000000	125.000000	824.249984
34	325.000000	125.000000	837.999984
35	330.000000	130.000000	851.749984
36	335.000000	130.000000	865.749984
37	340.000000	130.000000	879.999984
38	345.000000	135.000000	894.249984
39	350.000000	135.000000	908.999984
40	355.000000	140.000000	923.749984
41	360.000000	140.000000	938.749984
42	365.000000	140.000000	953.999984
43	370.000000	145.000000	969.249984
44	375.000000	150.000000	984.999984
45	380.000000	150.000000	1000.999984
46	385.000000	155.000000	1017.249984
47	390.000000	160.000000	1033.999984
48	395.000000	160.000000	1050.999984
49	400.000000	165.000000	1068.249984
50	405.000000	170.000000	1085.999984
51	410.000000	170.000000	1103.999984
52	415.000000	175.000000	1122.249984
53	420.000000	180.000000	1140.999984
54	425.000000	180.000000	1159.999984
55	430.000000	185.000000	1179.249984
56	435.000000	190.000000	1198.999984
57	440.000000	190.000000	1218.999984
58	445.000000	195.000000	1239.249984
59	450.000000	200.000000	1259.999984
60	455.000000	200.000000	1280.999984
61	460.000000	200.000000	1302.999984
62	465.000000	200.000000	1325.999984
63	470.000000	200.000000	1349.999984
64	475.000000	200.000000	1374.999984
65	480.000000	200.000000	1400.999984
66	485.000000	200.000000	1427.999984
67	490.000000	200.000000	1455.999968
68	495.000000	200.000000	1484.999968
69	500.000000	200.000000	1514.999968

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OPTIMUM COST FOR 4 PLANTS

I	DEMAND	Z	OP.COST

1	175.000000	15.000000	558.349984
2	180.000000	15.000000	564.099984
3	190.000000	15.000000	576.599984
4	200.000000	15.000000	590.099984
5	220.000000	15.000000	620.099984
6	250.000000	15.000000	672.599984
7	280.000000	15.000000	734.099984
8	300.000000	25.000000	778.999984
9	320.000000	35.000000	826.099984
10	340.000000	45.000000	875.399976
11	360.000000	50.000000	926.749976
12	375.000000	55.000000	966.399976
13	390.000000	60.000000	1007.349968
14	400.000000	65.000000	1035.099968
15	410.000000	65.000000	1063.599968
16	425.000000	65.000000	1108.099968
17	440.000000	65.000000	1154.349968
18	450.000000	65.000000	1186.599968
19	475.000000	65.000000	1273.349968
20	500.000000	65.000000	1368.349968
21	525.000000	65.000000	1472.349968
22	540.000000	65.000000	1544.349968
23	550.000000	65.000000	1597.349968
24	565.000000	65.000000	1684.349952

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VALUE OF GAMMA 1HYDEL PLANT
— 0.1000
QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT
28823.00

1	175.000000	15.000000	562.884984
2	180.000000	15.000000	568.634984
3	190.000000	15.000000	581.134984
4	200.000000	15.000000	594.634984
5	220.000000	15.000000	624.634984
6	250.000000	15.000000	677.134984
7	280.000000	15.000000	738.634984
8	300.000000	15.000000	784.634984
9	320.000000	25.000000	833.774976
10	340.000000	30.000000	885.089976
11	360.000000	40.000000	938.209976
12	375.000000	45.000000	979.514976
13	390.000000	50.000000	1021.899976
14	400.000000	55.000000	1050.614968

15	410.000000	55.000000	1080.114960
16	425.000000	65.000000	1125.034960
17	440.000000	65.000000	1171.284960
18	450.000000	65.000000	1203.534960
19	475.000000	65.000000	1290.284960
20	500.000000	65.000000	1385.284960
21	525.000000	65.000000	1489.284960
22	540.000000	65.000000	1561.284960
23	550.000000	65.000000	1614.284960
24	565.000000	65.000000	1701.284944

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VALUE OF GAMMA 1HYDEL PLANT

0.1100

QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT

26671.50

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1	175.000000	15.000000	567.419984
2	180.000000	15.000000	573.169984
3	190.000000	15.000000	585.669984
4	200.000000	15.000000	599.169984
5	220.000000	15.000000	629.169984
6	250.000000	15.000000	681.669984
7	280.000000	15.000000	743.169984
8	300.000000	15.000000	789.169984
9	320.000000	15.000000	839.169976
10	340.000000	20.000000	891.929976
11	360.000000	30.000000	947.029968
12	375.000000	30.000000	989.529968
13	390.000000	40.000000	1033.319976
14	400.000000	40.000000	1063.069976
15	410.000000	45.000000	1093.379968
16	425.000000	50.000000	1139.799968
17	440.000000	60.000000	1187.719968
18	450.000000	65.000000	1220.469968
19	475.000000	65.000000	1307.219968
20	500.000000	65.000000	1402.219968
21	525.000000	65.000000	1506.219968
22	540.000000	65.000000	1578.219968
23	550.000000	65.000000	1631.219968
24	565.000000	65.000000	1718.219952

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VALUE OF GAMMA 1HYDEL PLANT

0.1200

QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT

24167.50

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1	175.000000	15.000000	571.954984
2	180.000000	15.000000	577.704984
3	190.000000	15.000000	590.204984
4	200.000000	15.000000	603.704984
5	220.000000	15.000000	633.704984
6	250.000000	15.000000	686.204984
7	280.000000	15.000000	747.704984
8	300.000000	15.000000	793.704984
9	320.000000	15.000000	843.704976
10	340.000000	15.000000	896.954976
11	360.000000	15.000000	953.204976
12	375.000000	20.000000	997.069976
13	390.000000	30.000000	1041.969968
14	400.000000	30.000000	1072.469968
15	410.000000	35.000000	1103.754960
16	425.000000	40.000000	1151.929968
17	440.000000	50.000000	1201.699968
18	450.000000	55.000000	1235.794960
19	475.000000	65.000000	1324.154960
20	500.000000	65.000000	1419.154960
21	525.000000	65.000000	1523.154960
22	540.000000	65.000000	1595.154960
23	550.000000	65.000000	1648.154960
24	565.000000	65.000000	1735.154944

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VALUE OF GAMMA 1HYDEL PLANT
0.1300
QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT
21962.50

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1	175.000000	15.000000	576.489984
2	180.000000	15.000000	582.239984
3	190.000000	15.000000	594.739984
4	200.000000	15.000000	608.239984
5	220.000000	15.000000	638.239984
6	250.000000	15.000000	690.739984
7	280.000000	15.000000	752.239984
8	300.000000	15.000000	798.239984
9	320.000000	15.000000	848.239976
10	340.000000	15.000000	901.489976
11	360.000000	15.000000	957.739976
12	375.000000	15.000000	1002.239976
13	390.000000	20.000000	1048.209976
14	400.000000	25.000000	1079.849968

15	410.000000	25.000000	1112.099968
16	425.000000	35.000000	1161.889968
17	440.000000	40.000000	1213.289968
18	450.000000	45.000000	1248.609968
19	475.000000	55.000000	1340.009968
20	500.000000	65.000000	1436.089952
21	525.000000	65.000000	1540.089952
22	540.000000	65.000000	1612.089952
23	550.000000	65.000000	1665.089952
24	565.000000	65.000000	1752.089936

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VALUE OF GAMMA 1HYDEL PLANT
0.1400
QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT
20361.00

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1	175.000000	15.000000	581.024984
2	180.000000	15.000000	586.774984
3	190.000000	15.000000	599.274984
4	200.000000	15.000000	612.774984
5	220.000000	15.000000	642.774984
6	250.000000	15.000000	695.274984
7	280.000000	15.000000	756.774984
8	300.000000	15.000000	802.774984
9	320.000000	15.000000	852.774976
10	340.000000	15.000000	906.024976
11	360.000000	15.000000	962.274976
12	375.000000	15.000000	1006.774976
13	390.000000	15.000000	1053.024976
14	400.000000	15.000000	1085.274976
15	410.000000	20.000000	1118.599968
16	425.000000	25.000000	1169.874960
17	440.000000	35.000000	1223.024976
18	450.000000	35.000000	1259.274976
19	475.000000	45.000000	1353.474960
20	500.000000	60.000000	1452.399968
21	525.000000	65.000000	1557.024976
22	540.000000	65.000000	1629.024976
23	550.000000	65.000000	1682.024976
24	565.000000	65.000000	1769.024960

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VALUE OF GAMMA 1HYDEL PLANT
0.1500
QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT
18909.00

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1	175.000000	15.000000	585.559984
2	180.000000	15.000000	591.309984
3	190.000000	15.000000	603.809984
4	200.000000	15.000000	617.309984
5	220.000000	15.000000	647.309984
6	250.000000	15.000000	699.809984
7	280.000000	15.000000	761.309984
8	300.000000	15.000000	807.309984
9	320.000000	15.000000	857.309976
10	340.000000	15.000000	910.559976
11	360.000000	15.000000	966.809976
12	375.000000	15.000000	1011.309976
13	390.000000	15.000000	1057.559976
14	400.000000	15.000000	1089.809968
15	410.000000	15.000000	1123.559968
16	425.000000	20.000000	1176.239968
17	440.000000	25.000000	1230.649968
18	450.000000	30.000000	1268.039968
19	475.000000	40.000000	1364.759968
20	500.000000	50.000000	1466.399968
21	525.000000	65.000000	1573.959968
22	540.000000	65.000000	1645.959968
23	550.000000	65.000000	1698.959968
24	565.000000	65.000000	1785.959952

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VALUE OF GAMMA 1HYDFL PLANT

0.1600

QUANTITY OF WATER UTILIZED BY 1HYDEL PLANT

17938.00

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APPENDIX - B

DETERMINATION OF LOSS COEFFICIENTS

The transmission loss formula expressing the total transmission losses as a function of source power was first presented by George[1] in 1943. This formula was of the following form

$$P_T = B_{11} P_1^2 + B_{22} P_2^2 + \ldots\ldots + B_{nn} P_n^2 + 2B_{12} P_1 P_2$$

$$+ 2B_1 B_3 P_1 P_3 + \ldots\ldots + 2B_{mn} P_m P_n$$

$$= \sum_m \sum_n P_m B_{mn} P_n$$

where P_m and P_n = source power

and B_{mn} = transmission loss coefficient.

The determination of loss coefficients was based on lengthy procedure requiring considerable amount of time.

The application of network analyser to determine simil loss formula was developed by Ward, Eaton and Hale[7] in 1950. This method has also physical limitations in its application large systems. However an improved method for the calculatic of the transmission loss formula with the help of network analyser was developed by Stagg and Kirchmayer[8] in 1951.

The first paper to calculate the transmission loss formula for the calculation of losses by the application of digital computer was presented by Glimn, Habermann, Kirchmayer and Stagg[2] in the year 1953. Later on Brownlee[9] developed the method for expressing transmission losses as a function of generator voltage, angle and ratio X/R of the transmission circuit and the same was published in the year 1954. Loss formula involving linear terms and a constant term, in addition to the quadratic term, had been developed later by Early, Watson and Smith[10] and also by Early and Watson[11] in the year 1955. The transmission loss formula then becomes of the form :

$$P_L = \sum_m \sum_n P_m B_{mn} P_n + \sum B_{no} P_n + B_{oo}$$

This form of loss formula is very flexible with regards to assumptions regarding the manner in which each individual load varies with the total load i.e.,

$$I_L(1) = l_1 I_L \longrightarrow l_1 = \frac{I_{L(1)}}{I_L}$$

$$I_L(2) = l_2 I_L \longrightarrow l_2 = \frac{I_{L(2)}}{I_L}$$

and so on.

Where $I_L(1)$, $I_{L(2)}$ = individual loads at buses 1,2...etc.

and I_L = total load = $I_{L(1)} + I_{L(2)} + \dots$

$$= \sum^{K} I_{L(i)}$$

where K is the number of buses and l_1, l_2 etc. are some complex number relating the variation of individual load with the total load.

Subsequently there had been lot of work in the determination of loss coefficient known as a B_{mn} coefficient, more efficiently and many papers have been published in this direction. A quick method for developing transmission loss formula was presented during the year 1960 by Despotovic[12]. Here the total losses are expressed in terms of source power and assumption is made that the angle of load current and generation currents are equal. Later on during December 1960, a method for the direct calculation of transmission loss formulae was developed by Kirchmayer and others[13]. This method directly determines the line impedance and the reactive characteristics of the sources. In addition, the computer also determines the factor 'S' and non-conforming loads. A new method for the determination of loss coefficient has been developed by Stevenson and Hill[14] during the year 1968. In this method, after the initial load flow studies, loads are changed by ±5% or ±10% and then keeping generation constant at all buses except slack bus, the new load flow studies are carried out. The voltage for all the buses are kept constant throughout.

However to develop a general formula for the transmission losses in terms of generator powers, we first of all carry out the load flow studies. From the load flow studies we obtain the voltage magnitude and angle and also active and reactive power for all buses. Then currents injected into the buses are

also known, i.e.

$$V_{BUS} = (Z_{BUS})\, I_{BUS}$$

After this the factor l_1, l_2 etc. as described earlier is calculated. However if some loads (i.e. load currents) do not vary in proportion to the total load current, these are called non-conforming loads. Such loads can be split up into conforming load and non-conforming load, the non-conforming load is taken as the negative generation. Then with the help of bus impedance matrix and the factor S, different transform matrices are calculated. Thus we finally determine the loss coefficients with the help of which the transmission losses in terms of source powers can be calculated.